

# The Physical Origins of the Meissner Effect and London Penetration Depth

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Based on the recent developed real-space theory of superconductivity (arXiv:0910.5511 and arXiv:1001.5067), we study the physical nature of the Meissner effect and London penetration depth in conventional and non-conventional superconductors. It is argued that they originate from an exactly the same reason of the real-space quasi-one-dimensional periodic dynamic charge stripes in the superconductors. The fundamental relationship between the London penetration depth and the superconducting electron density is qualitatively determined.

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**Introduction:** The Meissner Effect is the most fundamental and the most dazzling property of materials in the superconducting state [1]. To explain the Meissner effect that no magnetic field exists in the interior of a superconductor, the London brothers proposed two equations to describe the microscopic electric field  $\mathbf{E}$  and magnetic field  $\mathbf{B}$  within the superconductor [2]. The first London equation is in fact the simply Newton's second law for the superconducting electrons:

$$\frac{\partial \mathbf{J}_s}{\partial t} = \frac{n_s e^2}{m_e} \mathbf{E}, \quad (1)$$

where  $\mathbf{J}_s = n_s e \mathbf{v}_s$  is the supercurrent density,  $n_s$  is the superconducting electron density and  $\mathbf{v}_s$  is the superfluid velocity. Taking into account Maxwell's equation of  $\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$ , then the second London equation can be written as

$$\nabla \times \mathbf{J}_s = -\frac{n_s e^2}{m_e} \mathbf{B}. \quad (2)$$

The London equations (1) and (2) imply a characteristic length scale  $\lambda_L = \sqrt{m_e / \mu_0 n_s e^2}$ , the so-called London penetration depth, which gives the length scale over which an external applied field is exponentially suppressed. For most superconductors, the London penetration depth  $\lambda_L$  is on the order of 100 nm. In 1950, Ginzburg and Landau (GL) developed the order parameter phenomenological theory to describe the superconducting phenomena in the conventional superconductors [3]. They derived a temperature dependent penetration depth as  $\lambda(T) = \sqrt{m^* c^2 / 4\pi e^2 |\Psi|^2}$ , where  $\Psi$  is the superconducting order parameter. While it is important to note that although these theories may offer part of the answer to the problems of the Meissner effect and penetration-depth effect, the exact physical origin of these effects remains a mystery so far. In addition, the theoretical foundation of the conventional London equations is not solid enough [4], as we know, there is a certain intuitive logic to this approach.

In our opinion, the phenomenological GL theory may include more important and rigorous information than the later-developed microscopic BCS theory [5]. The former was based on a coherent picture of all charge carriers, while the latter considered only the pairing behavior of the carriers. Physically, the pairing of the charge carriers in materials is an in-

dividual behavior characterized by pseudogap, while superconductivity is a collective behavior of many coherent charge carriers. Obviously, the BCS theory is lack of a convincing mechanism to ensure the coherence of the Cooper pairs. As more and more Cooper pairs, the backbone of the superconducting phenomenon, have now been observed in non-superconducting and insulation materials [6]. These interesting experiments strongly indicate that the BCS is likely to be physically incomplete.

In the past several years, we have published a series of articles which offer a new way of looking at the superconducting phenomena [7–10]. It is now very clear that pairing (pseudogap) is a real space Coulomb confinement effect within single unit cell [9, 10], while superconducting is related to the long-range order structures of electrons in real space [7, 8]. Surprisingly, no pairing and superconducting glues are needed in our scenarios where the direct Coulomb repulsive interactions between electrons can be naturally and completely suppressed.

In the present paper, we try to unveil the mystery of the Meissner effect and London penetration depth based on our previous works. We will show that the Meissner effect and London penetration depth are directly originated from the real-space periodic superconducting charge stripes.

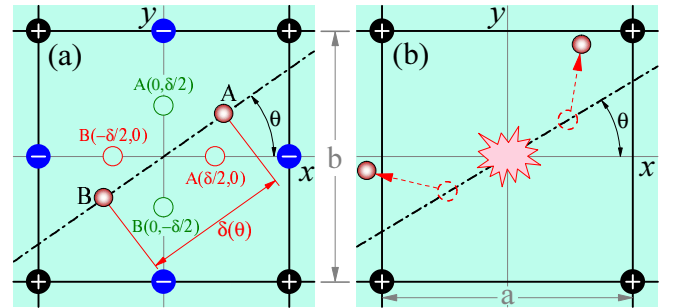


Figure 1: Two electrons (A and B) locate inside a single plaquette, (a) the hole-doped superconductor, there are four  $O^{1-}$  and four  $Cu^{2+}$  indicated by solid blue and black circles, respectively, the localized Cooper pair (pseudogap) may exist inside the plaquette due to the four nearest neighbor negative ions, (b) the conventional superconductor, where the localized Cooper pair is impossible to survive in the plaquette due to the Coulomb attractions by the positive ions.

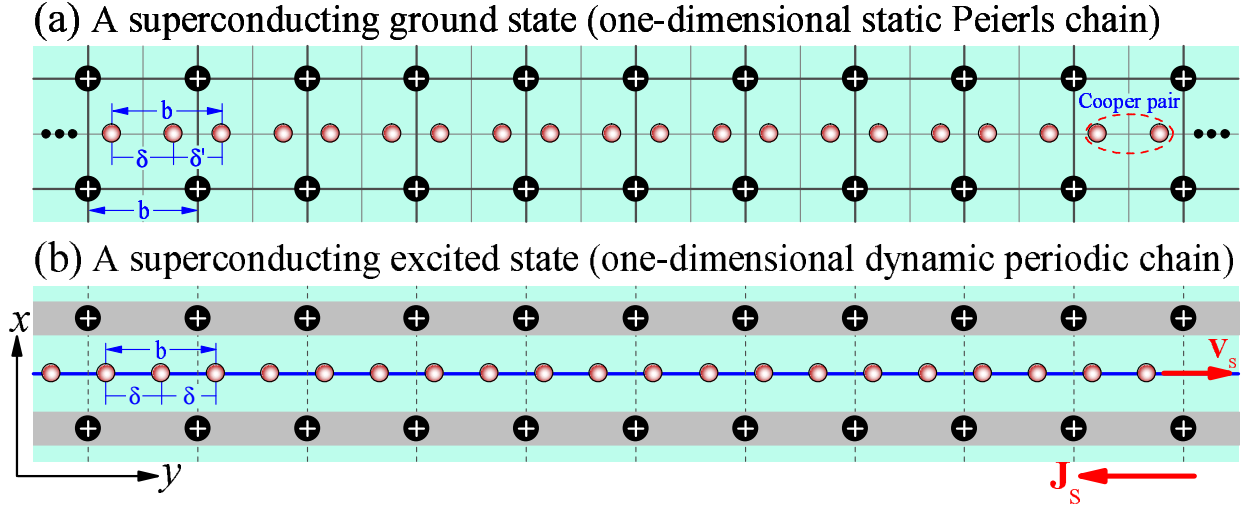


Figure 2: Two kinds of quantized one-dimensional charge stripes. (a). Without the driven of external fields, the electrons can arrange themselves into some static one-dimensional Peierls chains with two electrons (a real-space Cooper pair) inside one plaquette, we have proved analytically that the repulsive interactions among electrons have been completely suppressed with the appropriate  $\delta$  and  $\delta' = b - \delta$ . In the presence of the external fields, there will happen a charge order phase transition from the static Peierls charge stripe (superconducting ground state) to the dynamic periodic charge stripe (excited superconducting state). In this case, the electric field of the discrete positive ions are replaced by the uniform mean-fields which are regarded as domain walls and the electron-electron Coulomb repulsions can be naturally eliminated due to the symmetry of the stripe.

**Real Space Charge Stripes:** Normally, electrons repel each other according to Coulomb's law, they are attracted only to protons or positive ions. However, it is argued that strong electron-electron repulsions should be eliminated in some particular situations like superconductivity. How could the direct repulsive forces be completely suppressed in the superconductors?

As shown in Fig. 1, we have a new window on this critical question of what holds two electrons together [9, 10]. For the hole-doped cuprates, we argued that a localized hole-pair is a cluster of two electrons (a localized Cooper pair), four  $O^{1-}$  and four  $Cu^{2+}$  inside the Cu-O plane, as illustrated in Fig. 1(a). It is easy to prove that the direct and strong electron-electron repulsion can be entirely excluded if two electrons are located symmetrically at  $[A(\delta/2, 0), B(-\delta/2, 0)]$  in  $x$ -direction or  $[A(0, \delta/2), B(0, -\delta/2)]$  in  $y$ -direction within each copper-oxide unit with the Cooper-pair size  $\delta \approx 0.396b$  (when  $a = b$ ), as shown in Fig. 1(a). Moreover, the four nearest-neighbor electron- $O^{1-}$  repulsive interactions play the key role of the 'pairing glue' for the real-space localized Cooper pair. Hence, it should be noted that the existence of the negative ions is the most important condition for supporting the pseudogap phases in the materials (no limited to the superconductors) [7, 9]. This simple and reliable approach leads to the  $d$ -wave symmetry pseudogap behavior in the hole-doped cuprates. For the conventional superconductors of Fig. 1(b), the two electrons cannot maintain pairing in the proposed structure due to the direct electron-ion Coulomb attractions, as a result, these materials do not support the pseudogap's electronic state [9, 10].

It is now widely believed that the electron pairing (Cooper

pair) is not the most fundamental reason for the superconductivity [6, 11, 12]. The key to solve the superconducting puzzle lies in the problem of how can the Coulomb repulsions between pairs of electrons be overcome in favor of the superconductivity? In fact, the answer is very simple: symmetry and minimum energy principle. Generally, a superconductor is composed of two parts: the periodically arranged positive ions and the electrons (see Fig. 2), which only interact via the electromagnetic interaction. Based on the real space Coulomb confinement effect, we proved theoretically that the one-dimensional superconducting ground state (a static Peierls chain with  $v_s = 0$ ) and the excited

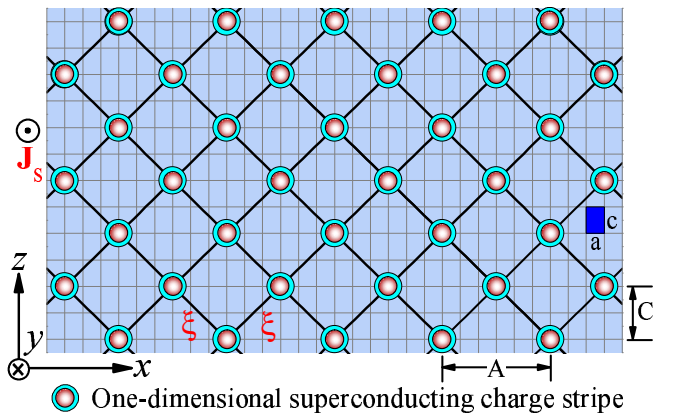


Figure 3: One of the stable structure of the superconducting vortex lattices (quasi-two-dimensional charge stripes), where the electron-electron repulsions among different stripes can also be canceled out because of the symmetry structure.

state (a dynamic periodic charge stripe with the electron-electron distance  $\delta = b/2$  and  $\mathbf{v}_s \neq 0$ ) can be symmetry and naturally formed inside one superconducting plane, as shown in Figs. 2(a) and (b) respectively. In our scenarios, all the superconducting electrons are assumed to be in a zero-stress state (the electron-electron repulsions are naturally suppressed) and can be regarded as the ‘inertial electrons’. The quasi-one-dimensional charge stripes of Fig. 2 may further self-assemble into some stable quasi-two-dimensional Wigner structures with trigonal or tetragonal symmetries [7, 8], see Fig. 3 as an example.

**Meissner Effect and London Penetration Depth:** We now firmly believe that all superconducting phenomena may come from an identical physical reason, which we think may be due to the ordered structures of the charge carriers in real space (see Fig. 2 and Fig. 3). In this section, we try to explain the observed facts of Meissner effect and London penetration depth in various superconductors using the suggested model.

In our framework, the so-called superconducting electron states are in fact the periodic arrays of aligned infinite straight wires (charge stripes). Suppose there are  $N$  dynamic charge stripes (each with a current  $I_0$ ) distributed evenly in a superconductor, then the magnetic field at any location (indicated ‘O’ in the Fig. 4) within the material can be expressed as:

$$\mathbf{B}_O = \sum_{i=1}^N \mathbf{B}_i, \quad (3)$$

where the value  $B_i = \mu_0 I_0 / 2\pi r_i$ , obviously, the magnetic field  $B_i$  will decrease rapidly with the increasing of  $r_i$ .

Strictly speaking, the magnetic field at any point should take into account all contributions of the charge stripes in the superconductor. But due to the rapid attenuation of the magnetic field, only a limited number of the charge stripes around the observation point will contribute to the magnetic field at point ‘O’ of Fig. 4. For a small radius  $r$  (see Fig. 4), only a few charge stripes are included and the total magnetic field generated by them is usually not equal to zero. As the radius  $r$  increases, more charge stripes produce magnetic fields at the circle center ‘O’, however, the total magnetic field will decay rapidly because of the symmetry of the intensive charge stripes inside the circle. This implies that there exists a critical radius  $r_c$  (or a critical charge stripes number  $N_c$  for a given supercurrent density  $\mathbf{J}_s$ ) that will make the total magnetic field  $\mathbf{B}_O = \sum_{i=1}^{N_c} \mathbf{B}_i \approx 0$ , naturally lead to the Meissner effect.

In the following, we will turn to a simple discussion of the London penetration depth. By relating the second London equation (2) to Maxwell’s equations, and we obtain

$$\nabla^2 \mathbf{B} = \frac{\mu_0 n_s e^2}{m_e} \mathbf{B} = \frac{1}{\lambda_L^2} \mathbf{B}. \quad (4)$$

For the simple one-dimensional ( $x$ ) geometry, then we can obtain the solution from Eq. (4), that is  $B(x) = B_0 e^{-x/\lambda_L}$ , or  $B(\lambda_L) = B_0/e$  indicates that the magnetic field at the surface of the superconductor will decay to  $B_0/e$  at a distance  $\lambda_L$

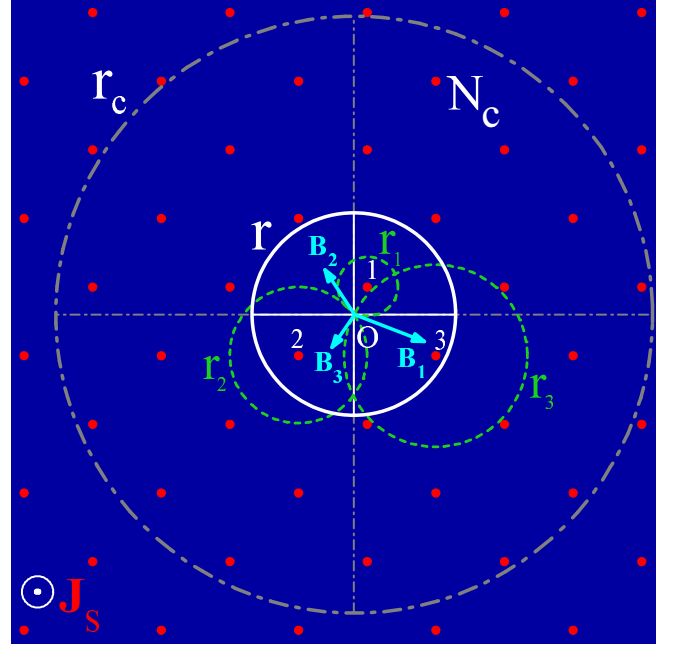


Figure 4: The explanation of the Meissner Effect. In our framework, the central feature in the superconductor is the real-space infinite charge stripes (indicated by red solid circles). Each stripe will generate a magnetic field at any location inside the superconductor (for example the ‘O’ point in the figure). Approximately, the net magnetic field of ‘O’ can be considered as the contribution from the limited number of charge stripes inside a critical circle  $r_c$ . As a result, the total magnetic field at point ‘O’ is expected to be zero due to the symmetry.

in the interior of the superconductor. Therefore, the London equations indeed lead to an exponential decay of the magnetic field within the superconductor. In our view, although the London equations provide a phenomenological description of the electromagnetic properties inside the superconductor, the most essential reason of this phenomenon is still unknown. This article demonstrates that the correct answer still lies in the nature of the charge orders of Fig. 2 and Fig. 3 in real space.

We assume that the shape of the studied superconductor is a cylinder with a cross section radius  $R$  and a superconducting current  $\mathbf{J}_s$  in  $-y$  direction, as shown in Fig. 5. As discussed above, there is an area ( $r \leq R - r_c$ , where  $r_c$  is the critical radius) inside the superconductor where the magnetic field is almost zero, as marked by the white dotted circle in the figure. When  $r > R - r_c$ , the critical circles no longer fall entirely within the superconductor (see the cases of C, D and E in Fig. 5), in these situations, the magnetic fields of the corresponding circle centers will not equal to zero. With  $r$  increasing from  $R - r_c$  to  $R$ , the net magnetic field will increase simultaneously from zero (the case B in Fig. 5) to the maximum value  $B_0$  (the case E in Fig. 5). Hence, the critical radius  $r_c$  can be regarded as the London penetration depth is associated with the superconducting electron density  $n_s$ . In our model, for a given superconductor, the structure of each superconducting

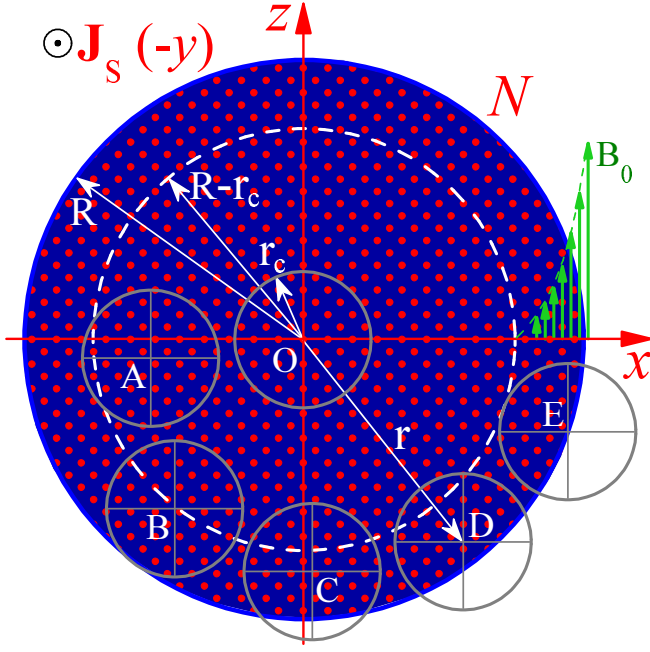


Figure 5: The explanation of the London penetration depth (see the text for details).

line [see Fig. 2 (b)] is independent of  $n_s$ , thus the number of the charge stripes  $N$  is proportional to  $n_s$ . Our discussions above are based on the assumption that the critical circle ( $r_c$ ) should cover approximately  $N_c$  charge stripes, and hence we have the following expression:

$$\pi r_c^2 n_s = \beta N_c = \text{Constant}, \quad (5)$$

where  $\beta$  is a constant. If we replace  $r_c$  with the penetration depth  $\lambda_L$  in Eq. (5), this immediately lead to the relationship  $\lambda_L \propto 1/\sqrt{n_s}$  which is in agreement with that of London equations. Notice that  $r_c$  is not equal to  $\lambda_L$ , since the critical parameter  $r_c$  is defined as  $B(r_c)/B_0 \rightarrow 0$  different from  $B(\lambda_L)/B_0 = 1/e \approx 37\%$  of the  $\lambda_L$ . Moreover, we think that the parameter  $\lambda_L$  does not describe accurately enough the magnetic flux penetration behavior in the superconductors. The real penetration depth is much larger than that determined by the London equations. Hence we introduce a new parameter  $\lambda(n)$  which is defined by  $\lambda(n) = n\lambda_L$  ( $n=1, 2, 3, 4, \dots$ ), then the attenuation of the magnetic field can be expressed as  $B[\lambda(n)]/B_0 = 1/e^n$ . When  $n=3$ , then  $\lambda(3) = 3\lambda_L$  and  $B[\lambda(3)]/B_0 = 1/e^3 \approx 5\%$  which is relatively more suitable for describing the experimental results.

It should be emphasized that our interpretations about the Meissner effect and London penetration depth are primarily based on the simple and ideal picture of the charge stripes described in Fig. 2 and Fig. 3, many factors such as temperature, crystal structures, the size and shape of the materials are not taken into consideration. In our theory the critical charge

stripe number  $N_c$  is a very important parameter which can be determined approximately by the experimental results. For the charge stripe structure of Fig. 3, each charge stripe occupies a area of  $\xi^2$  in  $xz$ -plane. According to our previous paper of the effective  $c$ -axis lattice constant theory of superconductivity [8], the stripe-stripe spacing  $\xi$  is closely related to the  $c$ -axis lattice constant of the superconductors. For conventional superconductors such as Pb of the cubic close-packed (CCP) structure with the lattice constants  $a = b = c = 0.495$  nm, it is likely that the charge carriers (electrons) form the superconducting charge stripe pattern of Fig. 3 with the minimum value of  $\xi^{Pb} = c/\sqrt{2} \approx 0.35$  nm. In addition, the experimental value of London penetration depth  $\lambda_L$  for Pb is about 40 nm, approximately, we have the critical radius  $r_c^{Pb} \approx \lambda(3) = 3\lambda_L = 120$  nm. Then the critical number  $N_c$  for Pb can be estimated as  $N_c = \pi (r_c^{Pb}/\xi^{Pb})^2 \approx 369000$ . One can find this critical  $N_c$  is a fairly large value, implying that the Meissner effect does not exist in the ultra-small superconductor grains. For the cuprate superconductors such as  $Hg_2Ba_2Ca_3Cu_4O_{10}$ , the stripe-stripe spacing  $\xi$  can reach about 1.9 nm which is about five times that of the conventional superconductors. As a result, the cuprate superconductors normally have a larger London penetration depth than the conventional superconductors.

**Brief Summary:** In this paper, the physical origins of the Meissner effect and London penetration depth observed in the superconducting materials have been studied. In our new framework, the Meissner effect and London penetration depth are the straightforward results of the real-space ordered charge stripes within the superconductors. We believe that the so-called complex phenomenon of superconductivity can be well understood from the simplest models and the most basic physical theories. Any artificial quasiparticles are in fact obstruct our understanding of phenomenon of superconductivity.

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